Analysis of Bridging Formulae in Transitional Regime

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Abstract. The most suitable method to compute aerodynamic forces of a spacecraft, at first stage of a design, relies on bridging formulae. There are two kinds of bridging formulae: global and local. The global formulae rely on knowledge of spacecraft aerodynamic force coefficients in continuum and in free molecular flow. The local formulae calculate the skin friction and pressure coefficients on the body surface; the global aerodynamic coefficients are then computed by integration. The aim of this work is to analyze the widely accepted local formulae by Potter and by Kotov. To this purpose, a simple body, like a sphere, has been preliminary considered and the results have been compared with those from the DSMC code DS2V. This comparison led to the corrections of the computation of the skin friction and pressure coefficients. These corrections have been applied to the Potter formula. On the other hand the original Kotov formula showed good results for the pressure coefficient at high altitudes. Therefore a merge of the corrected Potter formula and of the Kotov formula has been made. This methodology, called "new" bridging formula, has been successfully applied to sphere. The "new" formula has been also applied to EXPERT and ORION capsules, but it has to be pointed out that, in this application at low altitudes, a failure of the panel method starts to appear. Both local and global coefficients have been compared with the results by the DS2/3V codes. Finally, for these capsules, the global formula by Wilmoth has been also used by tuning the adjustable parameters.

Keywords: Local and global bridging formulae, DS2V and DS3V codes, EXPERT and ORION capsules, PACS: 47.45.-n, 47.40.Ki,

INTRODUCTION

In the past, several bridging formulae have been used to compute the aerodynamic forces of a spacecraft at first stage of a design (Phase A). There are two kinds of bridging formulae: global and local. The basic difference is that the global formulae rely on knowledge of the spacecraft aerodynamic force coefficients in continuum and free molecular flow, while the local formulae take directly into account the geometry of the vehicle and calculate pressure and skin friction distribution on the body surface. Then, the global aerodynamic coefficients are computed by integration of pressure and skin friction distributions on the body surface.

The aim of this work is to analyze the local formulae by Kotov, Lychkin, Reshetin and Schelkonogov [1] (here called Kotov formula) and by Potter and Peterson [2] (here called Potter formula), through the comparison of the results with the ones from the widely accepted DSMC code DS2V [3]. To this purpose, a simple body, like a sphere, has been considered. This comparison pointed out that the Kotov formula showed good match of the pressure coefficient at high altitudes, while some corrections were necessary at lower altitudes. These corrections have been applied to the Potter formula. For the skin friction coefficient, both formulae showed pretty good results at high altitudes but the results were not satisfactory in continuum low density regime. To obtain a satisfactory agreement with the DS2V results, changing the methodology of the skin friction coefficient computation was proper. Once again, the Potter formula has been chosen for the corrections. Therefore a "new" methodology (here called "new" bridging formula) has been developed. This relies, at low altitudes, on the use of the corrected Potter formula and, at higher altitudes, on the merge of the corrected Potter formula, for the computation of the skin friction coefficient, and of the Kotov formula, for the computation of the pressure coefficient.

The ultimate purpose of this work is to apply the "new" formula to more complex bodies, such as EXPERT and ORION capsules, along the re-entry path. These capsules have been chosen because characterized by completely different shapes. More specifically, ORION is "Apollo like" or sphere-cone shape, EXPERT is a blunted pyramidal shape, consisting of a body of revolution with an ellipse-clothoid-cone 2D longitudinal profile. For these capsules, in rarefied flow, pressure, skin friction and axial force coefficients, computed by the "new" bridging formulae, agree with the results by DS2V and DS3V [4] codes. On the opposite, in continuum low density regime, these coefficients over-estimate the DS2/3V results.

Finally, for these capsules, the global bridging formula by Wilmoth, Mitcheltree and Moss [5] (here called Wilmoth formula) has been also considered. By tuning the adjustable parameters, the axial force coefficient is in a very good agreement with the DS2/3V results in the whole transitional regime.

BRIDGING FORMULA BY KOTOV, LYCHKIN, RESHETIN AND SCHELKONOGOV

Kotov Lychkin, Reshetin and Schelkonogov [1] proposed a semi-empirical approximate method based on both numerical and experimental data for the calculations of aerodynamic characteristics of complex geometry bodies. The pressure coefficient (C_p) and skin friction coefficient (C_f) for a surface element with a local angle of incidence α_{loc} were presented in the following, general forms:

$$C_{p} = P_{0} + P_{1} \sin \alpha_{loc} + P_{2} \sin^{2} \alpha_{loc}$$
(1)
$$C_{f} = \tau_{0} \cos \alpha_{loc} + \tau_{1} \cos \alpha_{loc} \sin \alpha_{loc}$$
(2)

where P₀, P₁, P₂, τ_0 and τ_1 (called "regime coefficients") depend on similarity parameters, such as Reynolds number (Re₀= $\rho_{\infty}V_{\infty}L/\mu_0$, where μ_0 is the viscosity at the stagnation point), Mach number (M_{∞}), ratio of specific heats ($\gamma=c_p/c_v$), temperature ratio (t_w=T_w/T₀, where T_w is the wall temperature and T₀ is the stagnation temperature).

The equations, computing the "regime coefficients", are:

$$P_{0} = P_{0}^{id} + \left(P_{0}^{fm} - P_{0}^{id}\right) F_{P0} \quad (3) \qquad P_{1} = P_{1}^{fm}F_{P1} \quad (4) \qquad P_{2} = P_{2}^{id} + \left(P_{2}^{fm} - P_{2}^{id}\right) F_{P2} \quad (5)$$

$$\tau_{0} = \tau_{0}^{fm}F_{\tau 0} \quad (6) \qquad \tau_{1} = \tau_{1}^{fm}F_{\tau 1} \quad (7)$$

Superscripts "fm" and "id" refer to the free-molecular and ideal-continuum regimes, respectively. More specifically, the free molecular terms depend on the normal and tangential components of momentum, exchanged between gas and surface. The ideal-continuum terms depend on pressure coefficient at the stagnation point. A particular form of the functions $F_{\tau 0}$, $F_{\tau 1}$, F_{P0} , F_{P1} and F_{P2} is obtained by a semi-empirical procedure. This relies on the results from numerical calculations and experimental data about different bodies and at different test conditions.

BRIDGING FORMULA BY POTTER AND PETERSON

The values of skin friction (C_f) and pressure (C_p) coefficients are based on correlation of these quantities as computed for sphere by the DSMC method in transitional regime:

Skin friction. Is it possible to demonstrate [2] that the ratio between the skin friction coefficient in transitional regime and the skin friction coefficient in free molecular flow (C_{ffm}) can be correlated to the Z parameter that reads:

$$Z = f(9) \frac{\left[M_{\infty}/\sqrt{Re_{\infty}}\right] (T_{\infty}/T_{w})^{(1-\omega)/2} (80H_{w}/H_{0})^{y}}{\sin 9}$$
(8)

where: $y = V^{2.7} / (V^{3.1} + 180)$, $V = M_{\infty} / \sqrt{Re_{\infty}}$, $f(\theta)$ is a function correlating the DSMC data, for sphere $f(\theta)=1+\sin\theta$ and θ is the angle between the local surface normal and the free stream velocity.

Potter and Peterson computed C_f by a DSMC code and C_{ffm} by the well known Maxwell equation [6]. They obtained two correlation equations for $9 \le 75$ deg:

if Z > 1 then
$$C_f / C_{ffm} = [0.24/(0.24 + Z^{-1.3})]^{1.25}$$
 (9) if Z ≤ 1 then $C_f / C_{ffm} = 0.1284Z$ (10)

In the interval 75<9≤90 deg., C_f/C_{ffm} is computed by linear interpolation between the value of C_f/C_{ffm} at 9=75 deg. by Eqs.9 or 10, and the value at 9=90 deg., by multiplying the right hand side of Eq.9 by the factor 1+887.5/(7.46+ $Z^{1.14}$)² if Z≥1, or multiplying the right hand side of Eq.10 by the factor 1+12 Z^2 if Z<1.

Pressure. For estimating the values of p/p_{∞} , Potter and Peterson correlated this ratio as a function of M_{∞}/Re_{∞} . More specifically:

if
$$p_i \le p_{fm}$$
 then $p/p_{fm} = 1 - (1 - p_i / p_{fm}) / [1 + (0.6 + 9)^4 (M_{\infty} / Re_{\infty})^{1/2}]$ (11)

if
$$p_i > p_{fm}$$
 then $p/p_{fm} = 1 + (p_i/p_{fm} - 1)/[1 + 0.6(M_{\infty}/Re_{\infty})^{1/2}]$ (12)

where p_i is the pressure corresponding to inviscid flow and p_{fm} is the free molecular pressure, computed by the well known Maxwell equation [6]. Pressure p_i is computed from the ratio p_i/p_{∞} that is approximated by a curve, fitting the results obtained by the method of characteristics for hypersonic flow over a sphere:

$$p_{i}/p_{\infty} = 1 + 1.895S_{\infty}^{2}(1 + 0.1919 - 2.1439^{2} + 1.5649^{3} - 0.3349^{4})$$
(13)

BRIDGING FORMULA BY WILMOTH, MITCHELTREE AND MOSS

The global bridging formula, proposed by Wilmoth, Mitcheltree and Moss [5] to compute the aerodynamic force coefficients, as per the axial force coefficient C_A , is:

$$C_{A} = C_{A,cont} + (C_{A,fm} - C_{A,cont}) \sin^{2}(\phi)$$
(14)

where: subscript "cont" is for continuum, $\phi = \pi(a_1+a_2\log_{10}Kn_{\infty})$, a_1 and a_2 are constants. These constants are determined by choosing the Knudsen numbers corresponding to continuum and free molecular limits. For example, choosing $Kn_{cont}=10^{-3}$ and $Kn_{fm}=10$ one obtains $a_1=3/8$ (=0.375) and $a_2=1/8$ (=0.175). Furthermore, as the constants a_1 and a_2 are simply adjustable parameters, proper values may be chosen giving the best overall description of the transitional flows when additional data are available. Expressions similar to Eq.14 can be used for other aerodynamic coefficients: lift, drag and so on.

DS2/3V CODES AND TEST CONDITIONS

DS2/3V are "sophisticated" [7] and advanced codes. In fact, both codes allow the user to evaluate the quality of a run in terms of the adequacy of the number of simulated molecules by the visualization of the ratio of the molecule mean collision separation (mcs) and the mean free path (λ), in the same cell; mcs/ λ should be less than unity everywhere in the computational domain. According to Bird [3, 4], the adequacy of the run is achieved when the maximum value of mcs/ λ is less than 0.2.

The DS2V computational domain was a rectangle in the meridian plane: i) x=2.4 m, y=1.5 m for SPHERE (diameter 1.6 m), ii) x=2.4 m, y=2.3 m for EXPERT (length 1.55 m, base diameter 0.918 m), iii) x=6 m, y=3.5 m for ORION (length 3.3 m, base diameter 2.51 m). The DS3V computational domain for EXPERT was a parallelepiped: x=2.4 m, y=2.3 m, z=1.1 m.

The number of simulated molecules was about 2.0×10^7 . This number of simulated molecules, for the DS2V runs at the most severe test conditions, in terms of altitude, provided an average value of mcs/ λ : of about 0.39 for SPHERE (70 km), of about 0.25 for EXPERT (69.8 km), of about 0.1 for ORION (85 km). For the DS3V runs (EXPERT), at the most severe test conditions (80.4 km), the average value of mcs/ λ was 1.1.

For all runs, the simulation time was longer than 25 times the time necessary to cross the computing region along the x direction at the free stream velocity: 7500 m/s for sphere, 5000 m/s for EXPERT and 7600 m/s for ORION, therefore this time was of the order of 10^{-4} s. This simulation time can be considered long enough for stabilizing all thermo-fluid-dynamic parameters.

The working gas was simulated air made up of 5 chemical species: O₂, N₂, O, N and NO in chemical nonequilibrium. A fully accommodate gas-surface interaction model was used. In agreement with Potter [2], the wall temperature of SPHERE was 350 K. While wall temperature of capsules was 300 K. Free stream thermodynamic parameters were provided by the U.S. Standard Atmosphere 1976.

ANALYSIS OF THE RESULTS

Sphere. The first stage of the analysis of the results is related to the pressure and skin friction coefficient distributions on a sphere. Figures 1a, b show, as typical examples, the pressure coefficient distributions (a) at h=75 km (Kn_{D∞} \cong 1.7×10⁻³) and the skin friction coefficient distribution (b) at h=100 km (Kn_{D∞} \cong 7.5×10⁻²). The profiles of C_p both by Kotov and by Potter show good agreement with the DS2V results. However, it has to be pointed out that the values of C_p from both formulae slightly overestimate the ones by DS2V. In fact the average values of C_p by Potter, Kotov and DS2V are 0.947, 0.937 and 0.909, respectively. This condition is verified also at other altitudes up to 95 km (Kn_{D∞} \cong 3.6×10⁻²). At higher altitudes, the Kotov formula provides a good match with DS2V, while the

Potter formula slightly DS2V. Therefore, underestimates the C_p computational correcting for $Kn_{D\infty} < 3.6 \times 10^{-2}$ is procedure proper. The Potter formula has been chosen for this correction, therefore, Eqs.11 and 12 have been modified and read:

if
$$p_i \leq p_{fm}$$
 then

$$p/p_{\rm fm} = 1 + \frac{(p_i/p_{\rm fm}\alpha - 1)}{1 + \beta (M_{\infty}/Re_{\infty})^{1/2}} \quad (15)$$

if $p_i > p_{fm}$ then

$$p/p_{fm} = 1 + \frac{(p_i/p_{fm}\alpha - 1)}{1 + \beta (M_{\infty}/Re_{\infty})^{1/2}}$$
 (16)

The profiles of C_f do not show a satisfactory agreement with DS2V. This mismatch decreases at high altitudes (h>100 km), but amplifies at lower altitudes. Therefore improvement to the computation of C_f is necessary. Once again the Potter formula has been selected for the corrections. These are related to: i) exponents in Eq.9; these have been put at -1.6 and 0.85 instead of -1.3 and 1.25. ii) Ratio Cf/Cffm; this has been correlated with parameter Z* instead of Z. Z* reads:

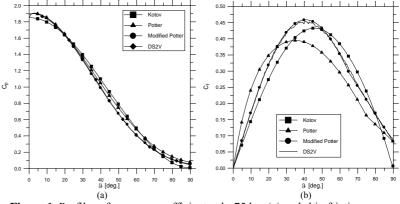


Figure 1. Profiles of pressure coefficient at h=75 km (a) and skin friction coefficients h=100 (b) on sphere

where: $\alpha = 0.8$, $\beta = 1$ if $Kn_{D\infty} \le 1.1 \times 10^{-3}$; $\alpha = 0.9$, $\beta = 5$ if $1.1 \times 10^{-3} < Kn_{D\infty} < 5.2 \times 10^{-3}$; $\alpha = 0.8$, $\beta = 10$ if $Kn_{D\infty} \ge 5.2 \times 10^{-3}$.

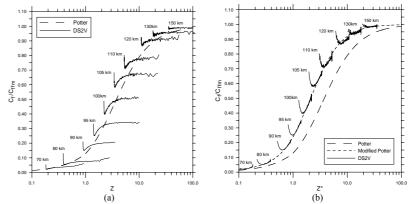


Figure 2. Correlation curves of friction coefficients from the Potter formula (a) and modified Potter formula (b), compared with the DS2V results

$$Z^* = \left[M_{\infty} / \sqrt{Re_{\infty}} \right] (T_{\infty} / T_{w})^{(1-\omega)/2} (1 + \cos \vartheta) (80 H_{w} / H_{0})^{y}$$
(17)

this parameter has been obtained by using the correlation function $f(9)=(1+\cos 9)\sin 9$ instead of $f(9)=(1+\sin 9)$. This new function f(9) has been obtained correlating new DSMC data, in-house computed. iv) The switch value; this has been put at 1.56 instead of 1. Finally, the modified Eqs.9 and 10 read:

if
$$Z^* > 1.56$$
 then $C_f / C_{ffm} = \left[0.24 / \left(0.24 + (Z^*)^{-1.6} \right) \right]^{0.85}$ (18)

if
$$Z^* \le 1.56$$
 then $C_f / C_{ffm} = 0.0026 + 0.1392(Z^*) + 0.1480(Z^*)^2 - 0.0523(Z^*)^3 + 0.0008(Z^*)^4$ (19)

Exponents in Eq.18 and coefficients of the polynomial (Eq.19) have been fixed by interpolating the values of C_f/C_{ffm} computed by DS2V.

In Figs.2a, b the results of the modified and of the original Potter formulae are compared with DS2V. The agreement of the modified Potter formula with DS2V is better than the one from the original formula. It has to be pointed out that the mismatch of the modified formula and DS2V, at each altitudes for 9>65 deg., has been overcome by a linear interpolation between the values of C_f/C_{ffm} at 9=65 deg., computed by Eqs. 16 and 17, and the values at 9=90 deg., computed by the following equations:

if $Z^* \ge 1$ then

$$C_{f}/C_{ffm} = \left[0.24/(0.24 + (2Z^{*})^{-1.6})\right]^{0.85} (1 + 887.5/(7.46 + (2Z^{*})^{1.14})^{2}$$

if Z*<1 then

$$C_{\rm f}/C_{\rm ffm} = [0.0026 + 0.1392(Z^*) + 0.1480(Z^*)^2 - 0.0523(Z^*)^3 + 0.0008(Z^*)^4] \text{K}$$

where $K = 8 \pm 1.0078(Z^*, 0.38)$ if $0.38 < Z^* < 1.0$ or $K = 5.5 \pm 12.26(Z^*)^2$

where $K=8+1.0078(Z^*-0.38)$ if $0.38 \le Z^* \le 1.0$ or $K=5.5+12.26(Z^*-0.18)$ if $Z^* \le 0.38$.

Figure 1b shows the comparison, with DS2V, of the C_f profiles on sphere at 100 km from the modified Potter, the original Potter and the Kotov formulae. The improvement of the modified Potter formula is evident, similar results have been obtained also in the range of altitude 70-150 km.

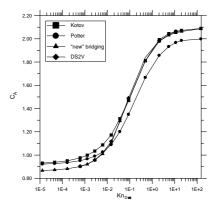


Figure 3. Profiles of axial force coefficient as function of Knudsen number

As reported in Fig.3, the corrections on C_p and C_f influence favorably the computation of C_A . In fact, the better agreement of the

values of C_A from the "new" bridging formula with the ones from DS2V, compared with the results from the original Potter and Kotov formulae, is well apparent.

Capsules. Figures 4a, b show the profiles of pressure and skin friction coefficients for ORION as a function of the curvilinear abscissa (s) at the altitude of 130 km (Kn_{D∞}=1.7). As expected, considering that $Kn_{D\infty} > 3.6 \times 10^{-2}$, the agreement of C_p by Kotov with DS2V is better than the one by Potter (Fig.4a). As for as the skin friction coefficient, the modified Potter and Kotov formulae are in excellent agreement with DS2V (Fig.4b). Figures 5a, b show the profiles of pressure and skin friction coefficients at the altitude of 90 km $(Kn_{D\infty}=0.0047);$ the bridging formulae are not able to evaluate satisfactory both Cp and Cf. This is probably due to the failure of the panel method, that increases with decreasing altitude.

Figures 6a, b show the profiles of pressure and skin friction coefficients for EXPERT at 104.5 km ($Kn_{D\infty}$ =0.37). Figures show an over prediction of pressure and skin friction coefficients on the flap. As already pointed out by Ivanov [8], the flaps are exposed to a flow that is very different from the free stream one, input to the bridging formulae.

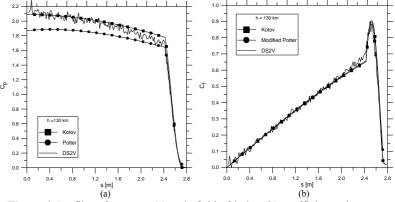


Figure 4. Profiles of pressure (a) and of skin friction (b) coefficients along ORION surface at h=130 km

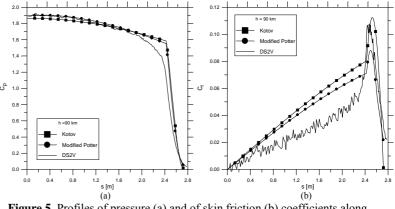
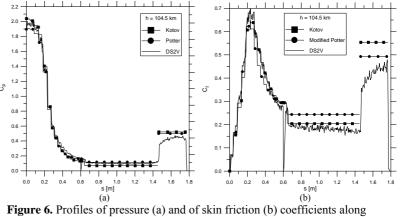


Figure 5. Profiles of pressure (a) and of skin friction (b) coefficients along ORION surface at h=90 km

For example, near the flap, the Mach number and the flow angle, computed by DS3V, range roughly between 2.5 and 3 and between 10 and 12 deg., respectively, while the free stream Mach number and the free stream flow angle, input to the bridging formulae, are 18 and 0 deg., respectively.

Figures 7a, b show the profiles of axial force coefficient as function of the Knudsen number for ORION and for EXPERT, respectively. The match is pretty good at high altitudes $(Kn_{D\infty} > 0.5)$ for ORION and $Kn_{D\infty} > 2.0$ for EXPERT), but at lower altitudes, the local bridging formulae do not match satisfactory DS2/3V; the percentage differences of CA from the "new" bridging formula with respect to DS2/3V are 5% and 19% for ORION and EXPERT. respectively. However, as already pointed out by Ivanov [8], an



EXPERT surface at h=104.5 km

uncertainty of 20% is acceptable in the Phase A of design of a re-entry vehicle. The agreement of the results from the Wilmoth formula with the ones from DS2/3V is excellent in the whole transitional regime. It has to be pointed out that, in this case, parameters a_1 and a_2 have been tuned for each capsule; for ORION $a_1=0.333$ and $a_2=0.143$, for EXPERT $a_1=0.353$ and $a_2=0.133$.

CONCLUSIONS

The local bridging formulae by Kotov and by Potter have been analyzed using a sphere. The comparison with the results from DS2V led to the corrections or modifications of the bridging formulae. For this purpose, the Potter formula has been chosen. A "new" bridging formula was obtained by the merge of the modified Potter formula and the Kotov formula. The "new" formula was used to compute the pressure and the skin friction distributions on two current capsules: EXPERT and ORION. The comparison of the local and global aerodynamic coefficients with the

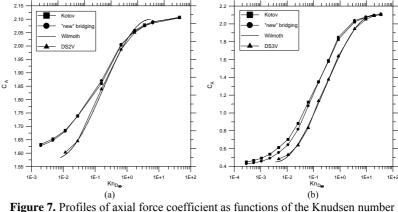


figure 7. Profiles of axial force coefficient as functions of the Knudsen number for ORION (a) and for EXPERT (b)

DS2/3V results verified that the "new" bridging formula is excellent at high altitudes but at low altitudes do not match satisfactory the DS2/3V results; this is probably due to a failure of the panel method.

Also the global bridging formula by Wilmoth was applied to these capsules. For this formula, thanks to proper values of the adjustable parameters, the axial force coefficient was in a very good agreement with the DS2/3V results in the whole transitional regime.

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